

Weak nonlinear oscillatory convection in a nonuniform heating porous medium with throughflow

Palle Kiran

Department of Applied Mathematics, School for Physical Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow-226025, India.

Email: kiran40p@gmail.com

ABSTRACT

A weak nonlinear stability analysis has been performed for the oscillatory mode of convection, heat and mass transports in terms of Nusselt, Sherwood numbers which are governed by the non-autonomous complex Ginzburg-Landau equation is calculated. The porous medium boundaries are heated sinusoidally with time. The basic state temperature is defined in terms steady and oscillatory parts, where steady part show nonlinear throughflow effect on the system. The generalized Darcy model is employed for the momentum equation. The disturbances of the flow are expanded in power series of amplitude of modulation, which is assumed to be small and employed using normal mode technics. The effect of vertical throughflow is found to stabilize or destabilize the system depending on its direction. The time relaxation parameter λ_1 has destabilizing effect, while time retardation parameter λ_2 has stabilizing effect on the system. Three types of modulations has been analyzed, and found that, OPM, LBMO cases are effective on heat/mass transfer than IPM case. The effects of amplitude and frequency of modulation on heat/mass transports have been analyzed and depicted graphically. The study establishes that the heat/mass transports can be controlled effectively by a mechanism that is external to the system. Further, it is found that better results may obtain for oscillatory mode of convection.

KEYWORDS

Weak nonlinear theory — Complex Ginzburg-Landau equation — Thermal modulation — Throughflow — Oscillatory convection.

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1. Introduction

Thermal instability in a horizontal fluid saturated porous medium has been extensively investigated and well documented by ([1]-[3]). An external controlling convective mechanism to a horizontal porous medium is very important in thermal and engineering science, geophysics and oil recovery processes etc. Temperature modulation is one of the effective mechanisms which controls the convective fluid flow in porous medium. Venezian [4], was the first person who performed a linear stability theory of Rayleigh-Bénard convection in a fluid layer for small amplitude of temperature modulation, deriving the correction in the critical Rayleigh number and he discussed the stability of fluid flow. One can see that for thermal modulation the conduction state temperature profile consisting of steady and oscillatory parts both. For the past 2 decades many researchers ([5]-[9]) studied only for stationary mode of nonlinear convection where growth rate of disturbances is zero, but, for oscillatory mode of

nonlinear convection very recently investigated by ([10]-[13]), where they have developed a new way of approach to determine an amplitude of convection while deriving non-autonomous complex Ginzburg-Landau equation.

Study of Double-diffusive convection is an important fluid dynamics property that involves motions driven by two different density gradients diffusing at different rates. Examples where this kind of convection is important are oceanography, modeling of solar ponds, lakes and under ground water, atmospheric pollution, chemical processes, laboratory experiments, electrochemistry, magma chambers and Sparks [14], formation of microstructure during the cooling of molten metals, fluid flows around shrouded heat dissipation fins, grain storage system, migration of moisture through air contained in fibrous insulations, crystal P growth, the dispersion of contaminants through water saturated soil, solidification of binary mixtures, and the underground disposal of nuclear wastes etc. Double-diffusive convection related to porous medium is of great practical importance in many branches of Science and Engineering, such as centrifugal Chemical engineering, Petroleum industry, Food engineering, filtration process, Geophysics, and Bio-mechanics. An excellent review of most of the findings has been given by ([15]-[20]).

The non-Newtonian fluid flow in either porous medium or fluid layer is of great practical applications in various areas of science, engineering and technology, chemical and nuclear industries, such as material processing, carbon dioxide geologic sequestration, bioengineering, and reservoir engineering. The importance of considering viscoelastic fluids is that, viscoelastic fluids exhibit unique patterns of instabilities such as the overstability that is not predicted or observed in Newtonian fluid case. Lowrie [21] investigated the viscoelastic behaviour of fluids is an important rheological process in the asthenosphere and in the deeper mantle. Herbert [22] and Green [23] were the first authors to study the oscillatory convection in an ordinary viscoelastic fluid of the Oldroyd type under the condition of infinitesimal disturbances. Vest and Arpacı [24] studied overstability in a viscoelastic fluid layer heated from below, and obtained the condition for the onset of thermally induced overstability. Bhatia and Steiner [25] investigated thermal instability in a rotating viscoelastic fluid layer, considering two sets of boundary conditions. They found destabilizing effect of rotation instead of stabilizing effect as in the case of Newtonian fluid. Rudraiah et al. [26, 27] studied the onset of oscillatory convection and modulation effect in a viscoelastic fluid saturating porous medium using different models. Kim et al. [28] studied thermal instability of viscoelastic fluids in porous media, conducted linear theory analysis for obtaining stability criteria for convective flow, and nonlinear oscillatory convection for heat transfer. They found that viscoelastic nature enhances heat transfer. Later on Yoon et al. [29] have studied the onset of oscillatory convection in a horizontal porous layer with viscoelastic fluid, considering linear theory. Other related studies to study thermal instability in viscoelastic fluids are given by Malashetty and Swamy [30], Tan and Masuoka [31], Sheu et al. [32], Wang and Tan [33]. Very recently Bhadauria and Kiran ([34]-[37]). investigated oscillatory mode of thermal instability using complex non-autonomous Ginzburg-Landau equation considering fluid or porous layer. They also discussed viscoelastic fluid properties along with gravity modulation while finding a better solution for oscillatory mode of convection.

Several geophysical and technological applications are involving non-isothermal flow of fluids through porous media, called throughflow. This flow alters the basic state temperature profile from linear to nonlinear with layer height, which in turn affects the stability of the system significantly. The effect of throughflow on the onset of convection in a horizontal porous layer has been studied by ([38]-[41]). Nield [42], Shivakumara [43] have justified a small amount of throughflow can have a destabilizing effect if the boundaries are of different types, and a physical explanation for the same has been given by them. Khalili and Shivakumara [44], the effect of throughflow and internal heat generation on the onset of convection in a porous medium is investigated. They have shown that throughflow destabilizes the system even if the boundaries are of the same type; a result which is not true in the absence of an internal heat source. The non-Darcian effects on convective instability in a porous medium with throughflow has been investigated in order to account for inertia and boundary effects by [45, 46]. Later on many researchers have investigated throughflow effects while considering different physical models some of them are given by ([47]-[53]).

Most of the studies on throughflow have been investigated using linear stability analysis and without modulation. However, if one wants to study heat/mass transfer along with the interaction of throughflow across the boundaries, it is essential to perform a weak nonlinear stability analysis under modulation. Further, to the best of author knowledge, not even a single study is available in which oscillatory mode of convection along with

throughflow has been investigated under thermal modulation. Therefore, the purpose of the present study is to investigate oscillatory mode of convection in a horizontal porous medium with viscoelastic fluid, using complex non-autonomous Ginzburg-Landau equation ([11, 13], [34]-[37]), under the effects of vertical throughflow and thermal modulation.

2. Mathematical formulation

An infinitely extended horizontal porous medium of depth ' d ' has been considered. The porous layer is homogeneous, isotropic, and saturated with viscoelastic fluid. The porous medium is heated and salted from below. Using the modified Darcy's model and employing the Boussinesq approximation for this system, the governing equations of flow is given by:

$$\nabla \cdot \vec{q} = 0, \tag{2.1}$$

$$\left(\overline{\lambda_1} \frac{\partial}{\partial t} + 1 \right) (-\nabla P + \rho \vec{g}) - \frac{\mu}{K} \left(\overline{\lambda_2} \frac{\partial}{\partial t} + 1 \right) \vec{q} = 0, \tag{2.2}$$

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa_T \nabla^2 T, \tag{2.3}$$

$$\frac{\partial S}{\partial t} + (\vec{q} \cdot \nabla) S = \kappa_S \nabla^2 S, \tag{2.4}$$

$$\rho = \rho_0 [1 - \alpha_T (T - T_0) + \beta_S (S - S_0)], \tag{2.5}$$

where \vec{q} is the fluid velocity, \vec{g} is acceleration due to gravity, ρ is density, K is the permeability of porous material, κ_T is the effective thermal diffusivity, κ_S is the effective thermal diffusivity in vertical direction, $\overline{\lambda_1}$ is stress relaxation time, $\overline{\lambda_2}$ is strain retardation time, α_T is the coefficient of thermal expansion and β_S is the coefficient of solute expansion. The externally imposed thermal and solutal boundary conditions are given by (Venezian [4] and Bhadauria and Kiran [13])

$$\begin{aligned} T &= T_0 + \frac{\Delta T}{2} [1 + \chi^2 \delta \cos(\Omega t)] && \text{at } z = 0 \\ &= T_0 - \frac{\Delta T}{2} [1 - \chi^2 \delta \cos(\Omega t + \phi)] && \text{at } z = d, \end{aligned} \tag{2.6}$$

$$\begin{aligned} S &= S_0 + \Delta S && \text{at } z = 0 \\ &= S_0 && \text{at } z = d. \end{aligned} \tag{2.7}$$

where ΔT and ΔS are the temperature and solute difference across the porous medium, χ is the smallness of amplitude of modulation, ϕ is the phase angle, δ , Ω are amplitude and frequency of temperature modulation.

3. Conduction state temperature and solutal fields

The basic state is assumed to be quiescent and the quantities in this state are given by

$$q_b = (0, 0, w_0), \quad \rho = \rho_b(z, t), \quad p = p_b(z, t), \quad T = T_b(z, t) \quad \text{and} \quad S = S_b(z, t). \tag{3.1}$$

Substituting Eq. (3.1) in Eqs. (2.1-2.5), we get the following relation which helps us to define hydrostatic pressure and temperature

$$\frac{\partial p_b}{\partial z} = \frac{\mu}{K} w_0 - \rho_b g, \tag{3.2}$$

$$\frac{\partial T_b}{\partial t} + w_0 \frac{\partial T_b}{\partial z} = \kappa_T \frac{\partial^2 T_b}{\partial z^2}, \quad (3.3)$$

$$w_0 \frac{\partial S_b}{\partial z} = \kappa_S \frac{\partial^2 S_b}{\partial z^2}, \quad (3.4)$$

$$\rho_b = \rho_0 [1 - \alpha_T (T_b - T_0) + \beta_S (S_b - S_0)]. \quad (3.5)$$

The solution of the Eqs. (3.3-3.4) subject to the boundary conditions Eq. (2.7), is given by:

$$T_b(z, t) = f(z) + \varepsilon^2 \delta \text{Re}[f_1(z, t)], \quad (3.6)$$

$$S_b = S_0 + \Delta S \frac{e^{(\frac{Pe}{\Gamma})z} - e^{(\frac{Pe}{\Gamma})}}{1 - e^{(\frac{Pe}{\Gamma})}}. \quad (3.7)$$

Here $f(z)$ is the steady part, while $f_1(z, t)$ is the oscillatory part of the basic temperature field will be defined in the next section, $Pe = \frac{w_0 d^2}{\kappa_T}$ is the Péclet number.

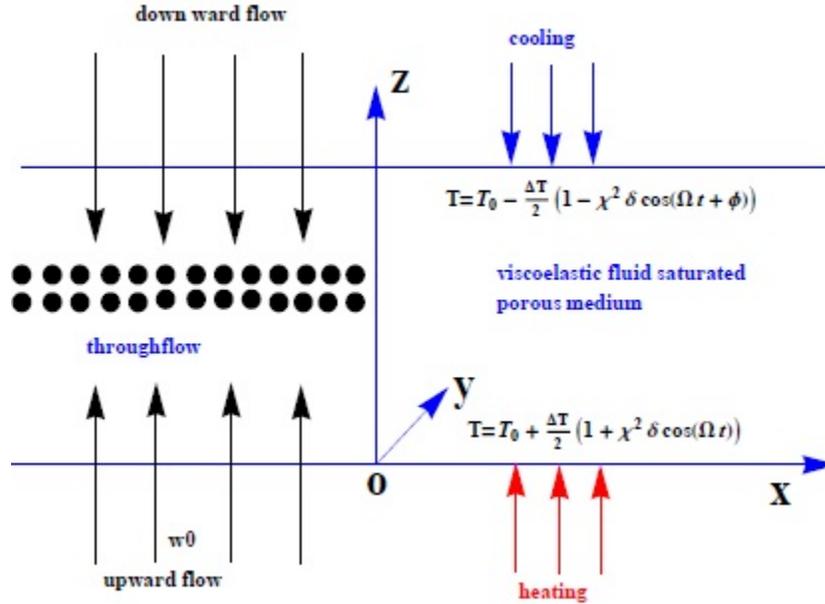


Figure 1. A sketch of the physical problem

4. Dimensionless governing equations

The finite amplitude perturbations on the basic state are superposed in the form,

$$\vec{q} = \vec{q}_b + \vec{q}', \quad \rho = \rho_b + \rho', \quad p = p_b + p', \quad T = T_b + T', \quad S = S_b + S'. \quad (4.1)$$

We introduce the Eq. (4.1) and the basic state temperature (Eq. 3.6) and solute (Eq. 3.7) fields in Eqs. (2.1-2.5), and then use the stream function ψ as $u' = \frac{\partial \psi}{\partial z}$, $w' = -\frac{\partial \psi}{\partial x}$, for two dimensional flow. The equations are then non-dimensionalized using the physical variables; $(x, y, z) = d(x^*, y^*, z^*)$, $t = \frac{d^2}{\kappa_T} t^*$, $\psi = \kappa_T \Psi^*$, $T' = \Delta T T^*$,

$S' = \Delta S S^*$, $\lambda_2 = \frac{\kappa_T \bar{\lambda}_2}{d^2}$, $\lambda_1 = \frac{\kappa_T \bar{\lambda}_1}{d^2}$ and $\Omega = \frac{\kappa_T}{d^2} \Omega^*$. The resulting non-dimensionalized system of equations can be expressed as (dropping the asterisk)

$$\left(\lambda_2 \frac{\partial}{\partial t} + 1 \right) \nabla^2 \psi + \left(\lambda_1 \frac{\partial}{\partial t} + 1 \right) \left(Ra \frac{\partial T}{\partial x} - Rs \frac{\partial S}{\partial x} \right) = 0, \quad (4.2)$$

$$-\frac{dT_b}{dz} \frac{\partial \psi}{\partial x} - \left(\nabla^2 - Pe \frac{\partial}{\partial z} \right) T = -\frac{\partial T}{\partial t} + \frac{\partial(\psi, T)}{\partial(x, z)}, \quad (4.3)$$

$$-\frac{dS_b}{dz} \frac{\partial \psi}{\partial x} - \left(\Gamma \nabla^2 - \frac{Pe}{\Gamma} \frac{\partial}{\partial z} \right) S = -\frac{\partial S}{\partial t} + \frac{\partial(\psi, S)}{\partial(x, z)}. \quad (4.4)$$

The non-dimensionalizing parameters in the above equations are: $Ra = \frac{\beta_T g \Delta T d K}{\nu \kappa_T}$ is thermal Rayleigh number,

$Rs = \frac{\beta_S g \Delta S d K}{\nu \kappa_S}$ is the solutal Rayleigh number, $\Gamma = \frac{\kappa_S}{\kappa_T}$ is diffusivity ratio and $\nu = \frac{\mu}{\rho_0}$ is kinematic viscosity. It is clear from the Eq. (4.3) that, throughflow and conduction profile of temperature, solute Eq. (4.4) affects the stability problem. The factor $\frac{\partial T_b}{\partial z}$, which is in Eq. (4.3) given by:

$$\frac{\partial T_b}{\partial z} = f'(z) + \varepsilon^2 \delta \text{Re}[f_1'(z, t)], \quad (4.5)$$

where $f' = \frac{Pe e^{Pe z}}{1 - e^{Pe}}$, $f_1'(z, t) = [B(\theta_2) e^{\theta_2 z} + B(-\theta_2) e^{-\theta_2 z}] e^{-i\Omega t}$, $B(\theta_2) = \frac{\theta_1 + \theta_2}{2} \frac{(e^{-i\phi} - e^{\theta_1 - \theta_2})}{e^{\theta_1 (e^{\theta_2} - e^{-\theta_2})}}$, $\theta_1 = \frac{Pe}{2}$, $\theta_2 = \frac{\sqrt{Pe^2 + 4\lambda_3^2}}{2}$ and $\lambda_3 = (1 - i) \sqrt{\frac{\Omega}{2}}$. The above system will be solved by considering the stress free and isothermal boundary conditions as given below

$$\psi = T = S = 0 \quad \text{at} \quad z = 0 \quad z = 1. \quad (4.6)$$

5. Derivation of complex Ginzburg-Landau equation

Introduce a small perturbation parameter χ that shows deviation from the critical state of onset of convection, the variables for a weak nonlinear state may be expanded as power series of χ as (Venezian [4] and Malkus and Veronis [55]):

$$\begin{aligned} Ra &= R_0 + \chi^2 R_2 + \chi^4 R_4 + \dots, \\ \psi &= \chi \psi_1 + \chi^2 \psi_2 + \chi^3 \psi_3 + \dots, \\ T &= \chi T_1 + \chi^2 T_2 + \chi^3 T_3 + \dots, \\ S &= \chi S_1 + \chi^2 S_2 + \chi^3 S_3 + \dots, \end{aligned} \quad (5.1)$$

where R_0 is the critical value of the Darcy-Rayleigh number at which the onset of convection takes place in the absence of thermal modulation. According to Kim et al. [28] and Bhadauria and Kiran ([34]-[37]) we introduce the fast time scale of time τ and the slow time scale of time s as $\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} + \chi^2 \frac{\partial}{\partial s}$.

Lowest order system

At first order the nonlinear terms in governing equations vanishes therefore, the first order problem reduces to the linear stability problem for overstability and we have:

$$\begin{bmatrix} \left(\lambda_2 \frac{\partial}{\partial \tau} + 1 \right) \nabla^2 & R_0 \left(\lambda_1 \frac{\partial}{\partial \tau} + 1 \right) \frac{\partial}{\partial x} & -Rs \left(\lambda_1 \frac{\partial}{\partial \tau} + 1 \right) \frac{\partial}{\partial x} \\ -\frac{dT_b}{dz} \frac{\partial}{\partial x} & \left(\frac{\partial}{\partial \tau} - \nabla^2 + Pe \frac{\partial}{\partial z} \right) & 0 \\ -\frac{dS_b}{dz} \frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial \tau} - \Gamma \nabla^2 + \frac{Pe}{\Gamma} \frac{\partial}{\partial z} \right) \end{bmatrix} \begin{bmatrix} \psi_1 \\ T_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (5.2)$$

The solution of the lowest order system subject to the boundary conditions, Eq. (4.6), is assumed to be

$$\psi_1 = (\mathbb{A}_1(s)e^{i\omega\tau} + \overline{\mathbb{A}_1}(s)e^{-i\omega\tau}) \sin ax \sin \pi z, \quad (5.3)$$

$$T_1 = (\mathbb{B}_1(s)e^{i\omega\tau} + \overline{\mathbb{B}_1}(s)e^{-i\omega\tau}) \cos ax \sin \pi z. \quad (5.4)$$

$$S_1 = (\mathbb{C}_1(s)e^{i\omega\tau} + \overline{\mathbb{C}_1}(s)e^{-i\omega\tau}) \cos ax \sin \pi z. \quad (5.5)$$

The undetermined amplitudes are functions of slow time scale, and are related by the following relation:

$$\mathbb{B}_1(s) = -\frac{4\pi^2 a}{(4\pi^2 + Pe^2)(c + i\omega)} \mathbb{A}_1(s), \quad (5.6)$$

$$\mathbb{C}_1(s) = -\frac{4\pi^2 a}{(4\pi^2 + (\frac{Pe}{\Gamma})^2)(\Gamma c + i\omega)} \mathbb{A}_1(s), \quad (5.7)$$

where $c = a^2 + \pi^2$. The critical Rayleigh number for oscillatory case is given by

$$R_0 = \frac{c(4\pi^2 + Pe^2) [c + \omega^2(\lambda_1 - \lambda_2) + \omega^2 \lambda_1 \lambda_2 c]}{4a^2 \pi^2 (1 + \omega^2 \lambda_1^2)} + \frac{Rs(4\pi^2 + Pe^2)(\Gamma c^2 + \omega^2)}{(4\pi^2 + (\frac{Pe}{\Gamma})^2)(\Gamma^2 c^2 + \omega^2)}. \quad (5.8)$$

Here calculate the corresponding critical wave number by minimizing critical Rayleigh number with respect to the wave number. The growth rate ω^2 can be obtained from:

$$a_1 \omega^4 + a_2 \omega^2 + a_3 = 0. \quad (5.9)$$

Observing closely on a_1, a_2, a_3 (are given in appendix) it reveals that the necessary condition for the occurrence of the oscillatory convection is that the following inequalities hold:

$$\lambda_1 > \lambda_2 \ \& \ \Gamma < 1. \quad (5.10)$$

Second order system

In this case the following relations in the case of nonlinear terms for temperature and solutal profile is obtained as

$$\psi_2 = 0, \quad (5.11)$$

$$\left(\frac{\partial}{\partial \tau} - \nabla^2\right) T_2 = \frac{\partial(\psi_1, T_1)}{\partial(x, z)}, \quad (5.12)$$

$$\left(\frac{\partial}{\partial \tau} - \Gamma \nabla^2\right) S_2 = \frac{\partial(\psi_1, S_1)}{\partial(x, z)}. \quad (5.13)$$

From the above relation, according to Kim et al. [28] and Bhadauria and Kiran ([34]-[37]), we can deduce that the velocity, temperature and solutal fields have the terms having frequency 2ω and independent of fast time scale. Thus, we write the second order temperature, solutal terms as follows:

$$T_2 = \{T_{20} + T_{22}e^{2i\omega\tau} + \overline{T}_{22}e^{-2i\omega\tau}\} \sin 2\pi z, \quad (5.14)$$

$$S_2 = \{S_{20} + S_{22}e^{2i\omega\tau} + \overline{S}_{22}e^{-2i\omega\tau}\} \sin 2\pi z, \quad (5.15)$$

where (T_{20}, T_{22}) and (S_{20}, S_{22}) are temperature and solutal fields having the terms having the frequency 2ω and independent of fast time scale, respectively. The second order solutions can be defined using T_2, S_2 in Eqs.(5.12)-(5.13). The horizontally averaged Nusselt, Sherwood numbers, for the oscillatory mode of convection is given by:

$$\text{Nu}(s) = 1 - \chi^2 \left(\frac{\partial T_2}{\partial z}\right)_{z=0}, \quad (5.16)$$

$$\text{Sh}(s) = 1 - \chi^2 \left(\frac{\partial S_2}{\partial z}\right)_{z=0}. \quad (5.17)$$

Third order system

At third order system obtain the following relation

$$\begin{bmatrix} \left(\lambda_2 \frac{\partial}{\partial \tau} + 1\right) \nabla^2 & R_0 \left(\lambda_1 \frac{\partial}{\partial \tau} + 1\right) \frac{\partial}{\partial x} & -Rs \left(\lambda_1 \frac{\partial}{\partial \tau} + 1\right) \frac{\partial}{\partial x} \\ -\frac{dT_b}{dz} \frac{\partial}{\partial x} & \left(\frac{\partial}{\partial \tau} - \nabla^2 + Pe \frac{\partial}{\partial z}\right) & 0 \\ -\frac{dS_b}{dz} \frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial \tau} - \Gamma \nabla^2 + \left(\frac{Pe}{\Gamma}\right) \frac{\partial}{\partial z}\right) \end{bmatrix} \begin{bmatrix} \psi_3 \\ T_3 \\ S_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \end{bmatrix}, \quad (5.18)$$

where the expressions for R_{31} , R_{32} and R_{33} are given in the appendix. The reader may note here, obtaining the solution at this stage is difficult and the following solvability condition can be used for existing the solution of the above system. Now under the solvability condition for the existence of third order solution, we obtain the following complex non-autonomous Ginzburg-Landau equation that describes the temporal variation of the amplitude $\mathbb{A}(s)$ of the convection cell

$$\frac{d\mathbb{A}(s)}{ds} - \gamma_1^{-1} F(s) \mathbb{A}(s) + \gamma_1^{-1} k_1 |\mathbb{A}(s)|^2 \mathbb{A}(s) = 0. \quad (5.19)$$

where the coefficients γ_1 , $F(s)$ and k_1 are given in the appendix. Writing $\mathbb{A}(s)$ in the phase-amplitude form, we get

$$\mathbb{A}(s) = |\mathbb{A}(s)| e^{i\theta}. \quad (5.20)$$

Now substituting the expression Eq.(5.20) in Eq.(5.19), we get the following equations for the amplitude $|\mathbb{A}(s)|$:

$$\frac{d|\mathbb{A}(s)|^2}{ds} - 2p_r |\mathbb{A}(s)|^2 + 2l_r |\mathbb{A}(s)|^4 = 0, \quad (5.21)$$

$$\frac{d(ph(\mathbb{A}(s)))}{ds} = p_i - l_i |\mathbb{A}(s)|^2, \quad (5.22)$$

where $\gamma_1^{-1} F(s) = p_r + ip_i$, $\gamma_1^{-1} k_1 = l_r + il_i$ and $ph(\cdot)$ represents the phase shift. For unmodulated system the amplitude is defined as

$$|\mathbb{A}(s)|^2 = \frac{\mathbb{A}_0^2}{\left(\frac{l_r}{p_r} \mathbb{A}_0^2 + \left(1 - \frac{l_r}{p_r} \mathbb{A}_0^2\right) e^{-2p_r s}\right)}, \quad (5.23)$$

where \mathbb{A}_0 is an initial amplitude. One can observe here from Eq. (5.21) for the current problem $l_r > 0$ and $Ra > R_0$ i.e. $p_r > 0$, the solution gives as $|\mathbb{A}| \sim \mathbb{A}_0 e^{p_r s}$ as $s \rightarrow -\infty$ and $\mathbb{A}_0 \rightarrow 0$ just as in the linear theory, but a new stable solution develops, $|\mathbb{A}| = \sqrt{\frac{p_r}{l_r}}$ as $s \rightarrow \infty$, whatever be the value of \mathbb{A}_0 . This is called supercritical pitch fork bifurcation, the base system being linearly unstable for $Ra > R_0$ but settling down as a new laminar flow. The steady state amplitude exists when R_0 takes positive values. The average values of Nusselt (\overline{Nu}), Sherwood (\overline{Sh}) numbers for understanding the effect of temperature modulation on heat and mass transports in a representative time interval i.e $(0, 2\pi)$, is obtained as

$$\overline{Nu} = \frac{1}{2\pi} \int_0^{2\pi} Nu(s) ds, \quad (5.24)$$

$$\overline{Sh} = \frac{1}{2\pi} \int_0^{2\pi} Sh(s) ds. \quad (5.25)$$

The temperature modulation effect enter through the factor $I_1(s)$, which determines whether the modulation amplifies or diminishes the amplitude of convection.

6. Results and discussions

The combined effect of temperature modulation and vertical throughflow has been investigated on double diffusive oscillatory convection in a viscoelastic fluid saturated porous medium. A weakly nonlinear stability analysis is considered to study heat and mass transfer while deriving a complex form of Ginzburg-Landau equation. The effect of temperature modulation is assumed to be of order $O(\chi^2)$, which means only small amplitude of modulation considered. Such an assumption will help us in obtaining the amplitude equation of convection in simple manner and is much easier to obtain than in the case of the Lorenz model. An externally imposed temperature field is important to regulate heat/mass transfer. The objective of this article is to consider such candidates, temperature modulation and vertical throughflow for either enhancing or inhabiting convective heat/mass transfer in the system.

Since the porous medium is assumed to be closely packed, the Darcy-model is employed. While plotting Nu/Sh versus slow time s , the effects of relaxational parameters, λ_1, λ_2 , the Péclet number Pe , the frequency Ω and the amplitude δ of modulation on heat/mass transport have presented in figures (2–5). It is observed that, the relation Eq. (5.10) leads to an interesting result; that for a horizontal porous medium heated from bellow; the oscillatory type of instability is possible only when the relaxation parameter λ_1 is greater than the retardation parameter λ_2 . Also, our study is to consider small amplitude of modulation on the heat/mass transport, therefore, the value of δ is considered around 0.1 Further, since the effect of low frequencies is maximum on the onset of convection as well as on the heat transport, therefore the modulation of temperature is assumed to be of low frequency. A small amount of throughflow in a particular direction is either to destabilizes or stabilize the system, hence, Pe values considered very small. The numerical results for Nu/Sh obtained from the expression in Eqs. (5.16-5.17) by solving the amplitude Eq. (5.21) have been presented in figures. (2–5). It is clear to see that, Eqs. (5.16-5.17) is conjunction with Eq. (5.21) that, Nu/Sh are the function of system parameters. The effect of each parameter on heat/mass transport is shown in figures (2–5) wherein the plots of Nusselt/Sherwood numbers versus time s are presented. It is found from the figures that the value of Nu/Sh starts with one and remains constant for quite some time, thus showing the conduction state initially. Then the values of Nu/Sh increases with time, thus showing the convection state and finally becomes constant on further increasing s thus achieving the steady state.

The effect of Péclet number Pe on heat/mass transfer is given in Fig. 2a and Fig. 3a, where downward throughflow is found to stabilize and upward throughflow destabilize the system. The oscillatory Rayleigh number R_0 increases with Pe , and independent of throughflow direction. This may be due to the fact that, throughflow is to confine significant thermal gradients to a thermal boundary layer at the boundary towards which the throughflow is directed. The effective length scale is thus smaller than the thickness of the porous layer. Hence the Rayleigh number will be much less than the actual value of Rayleigh number. Therefore, large values of Rayleigh number are needed for the onset of convection when the throughflow strength increases, which are the results obtained by Khalili and Shivakumar [44] in the case of free-free boundaries. The opposite results were obtained by Nield [42] in the case of fluid layer for small amount of throughflow. According to Shivakumara and Sureshkumar [48], the opposite effect may be due to the distortion of steady-state basic temperature distribution from linear to nonlinear by the throughflow. A measure of this is given by the basic state temperature and this can be interpreted as a rate of energy transfer into the disturbance by interaction of the perturbation convective motion with basic temperature gradient. The maximum temperature occurs at a place where the perturbed vertical velocity is high, and this leads to an increase in energy supply for destabilization. The effect of relaxational parameters λ_1 and λ_2 on heat/mass transfer is seen in Fig. 2b and Fig. 3b and Fig. 2c, Fig. 3c. It is clear that Nu/Sh decreases with decreasing λ_1 and increases with increasing λ_2 . This is due to the stabilizing effects of low values of λ_1 and high values of λ_2 , which are related with viscosity and elasticity of the viscoelastic fluids respectively. On the other hand, the amplitude of oscillation will decrease and the convective flow becomes stable with increment in λ_2 but decrement in λ_1 . Thus, it is confirmed that the elastic behavior of the non-Newtonian fluids leads to the oscillatory motions. The critical value of dimensionless frequency for marginally oscillatory modes is obtained from the relation, Eq. (5.9). One can notice here that the oscillatory frequency is dependent of viscoelastic parameters and independent of throughflow. These are compatible with the results of Kim et al. [28] and Rajib and Layek [55].

The effect of solutal Rayleigh number Rs (given in Fig. 2d and Fig. 3d) is to increase Nu/Sh hence heat

and mass transfer. Though the presence of a stabilizing gradient (solute concentration) prevent the onset of convection, the strong finite-amplitude motions, which exist for large Rayleigh numbers, tend to mix the solute and redistribute it so that the interior layers of the fluid are more neutrally stratified. As a consequence, the inhibiting effect of the solute gradient is greatly reduced and hence fluid will convect more and increases heat and mass transfer. The effect of diffusivity ratio (Γ) is to diminish heat and mass transfer (given in Fig. 2e and Fig. 3e). These results are conforms the results obtained by Bhadauria and Kiran [12, 13]. The modulation effect in terms of amplitude and frequency is shown in Figs. (2-3)f,g. The effect of amplitude of modulation is to increase the magnitude of Nu/Sh, thus increasing heat/mass transport in the system given in Fig.2f and Fig.3f. An increment in the frequency of modulation decreases the magnitude of Nu/Sh, and so the effect of frequency of modulation on heat/mass transport diminishes (given in Fig. 2g and Fig.3g). At high frequency the effect of thermal modulation on thermal instability disappears altogether. This result agrees quite well with the linear theory results of Venezian [4], where the correction in the critical value of Rayleigh number due to thermal modulation becomes almost zero at high frequencies.

The above results given figures 2-3 are corresponding to OPM (out of phase modulation) case, while fixing the values of parameters as ($Pe=0.1, Rs=10, \lambda_1 = 0.4, \lambda_2 = 0.1, \Gamma = 0.8, \delta = 0.1, \Omega = 2.0$), each individual parameter effect has been shown. The results corresponding to IPM (in phase modulation) and LBMO (lower boundary modulation only) are given in Fig.4 and the comparison made. The comparison of various boundary modulation on heat/mass transfer given in Fig. 4a and Fig.4b.. In-phase modulation negligible on heat transfer, Out of phase and Lower boundary modulations are effective for heat transfer. The Fig. 4c and Fig.4d, show the comparison between stationary and oscillatory mode of convection, and found that, the oscillatory mode of convection increases heat/mass transfer rather than stationary mode of convection. These results are conform the results obtained by Bhadauria and Kiran ([34]-[37]).

The effect of modulation on mean Nusselt number depends on both the phase difference ϕ and frequency Ω of modulation than only on the choice of the small amplitude modulation. From the Fig.5 it is evident that for a given frequency of modulation there is a range of ϕ in which \bar{Nu} increases with increasing ϕ and another range \bar{Nu} may decreases. Thus, the suitable choices of Ω and ϕ heat transfer can be controlled well. Heat transfer can be regulated (enhanced or reduced) with the external mechanism of temperature modulation effectively. We also can observe our results in Fig. 5 are the results which are similar to Siddheshwar et al. [9, 11, 56]. It is clear that for temperature modulation, the boundary temperatures should not be in IPM case (synchronized), where the effect of modulation is negligible on heat transport. The similar results can be obtained for \bar{Sh} .

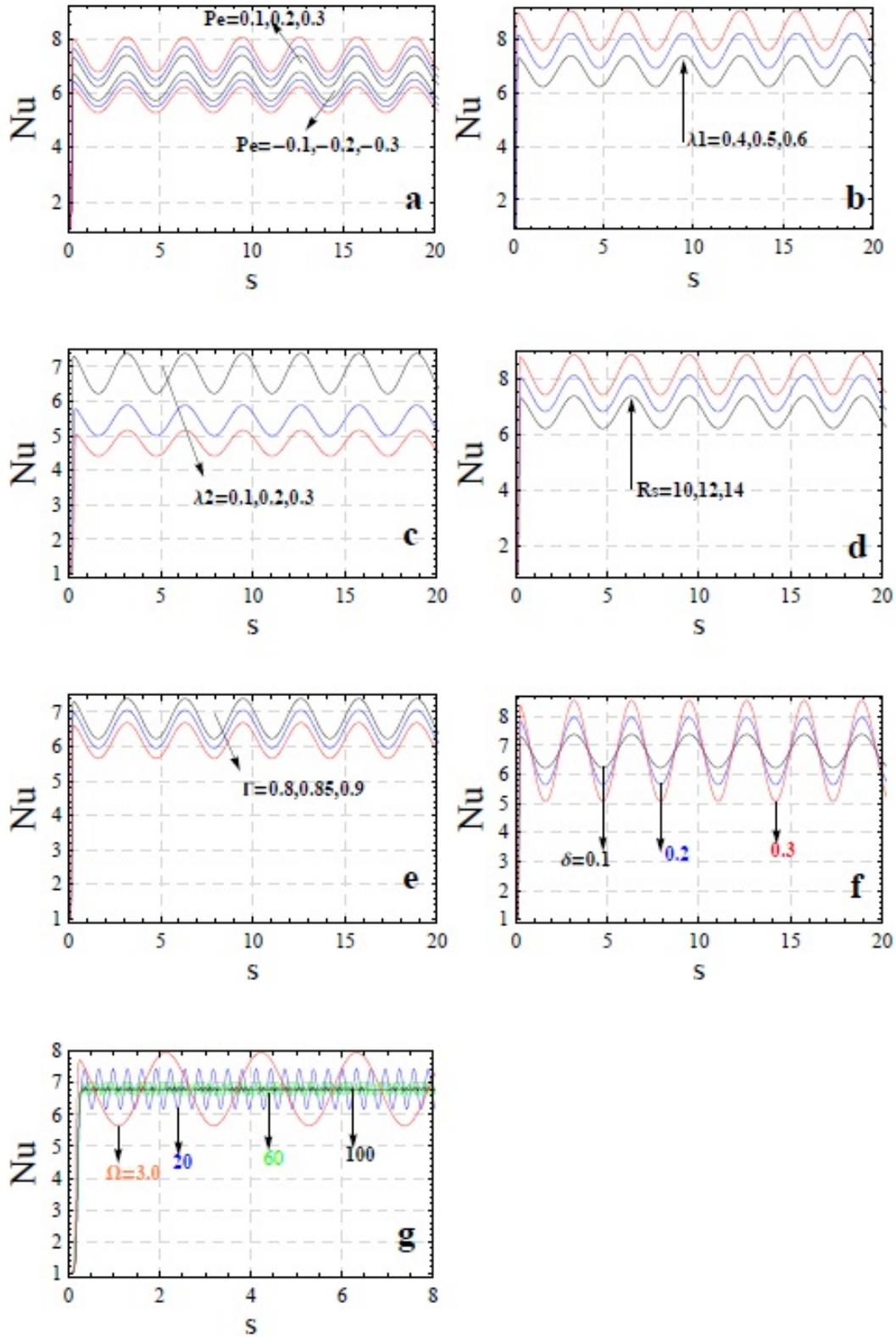


Figure 2. Variations in heat transfer with respect to the system parameters (OPM case)

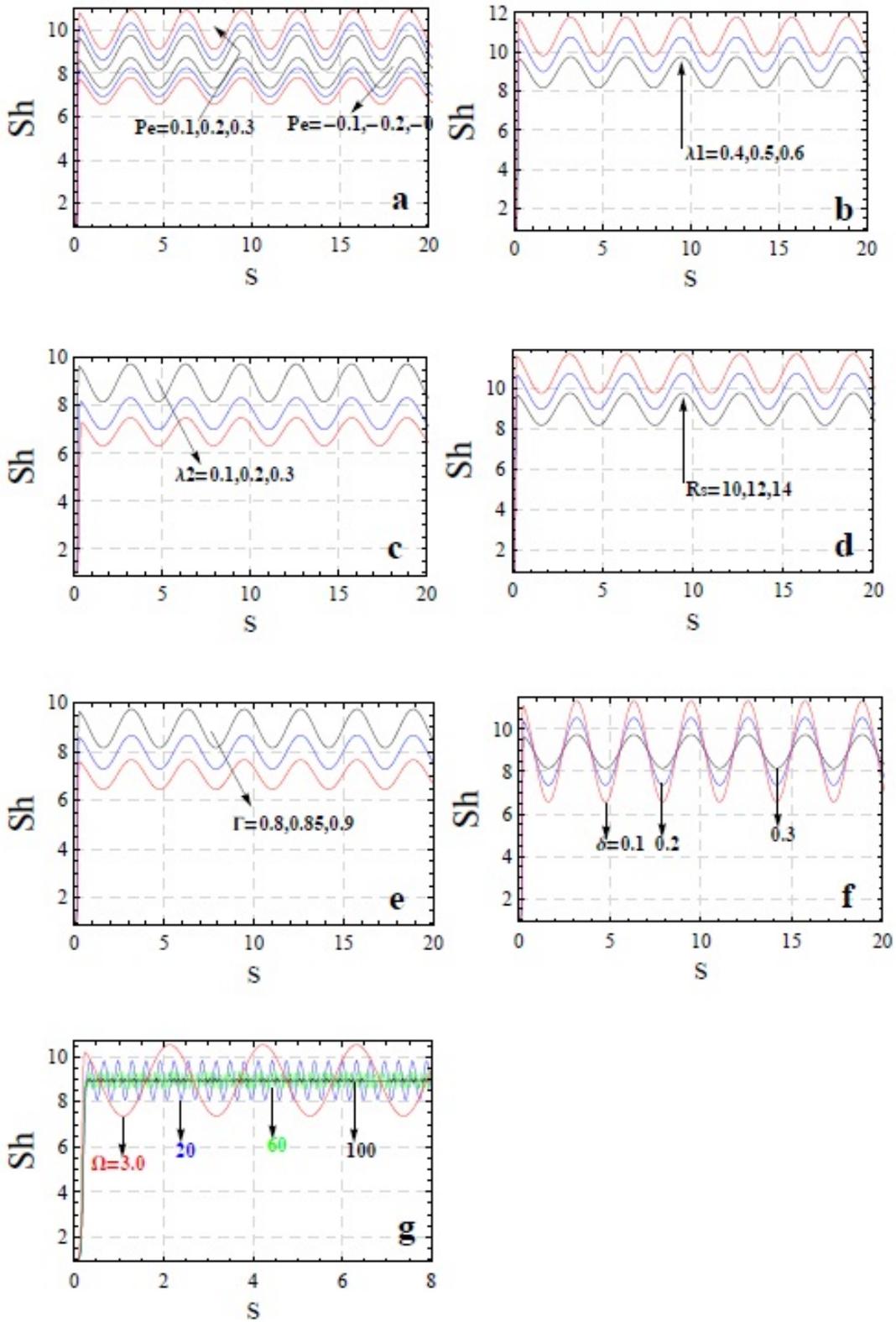


Figure 3. Variations in mass transfer with respect to the system parameters (OPM case)

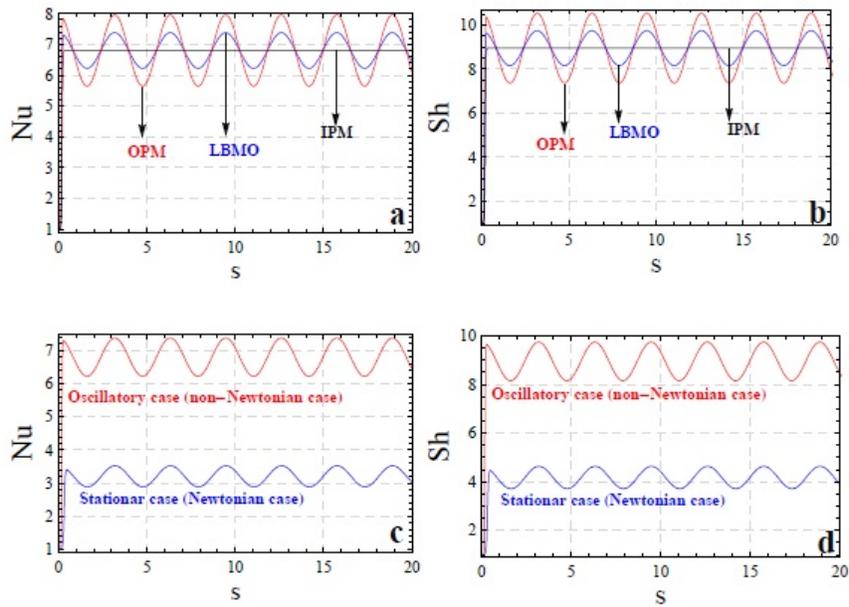


Figure 4. Comparison of different cases

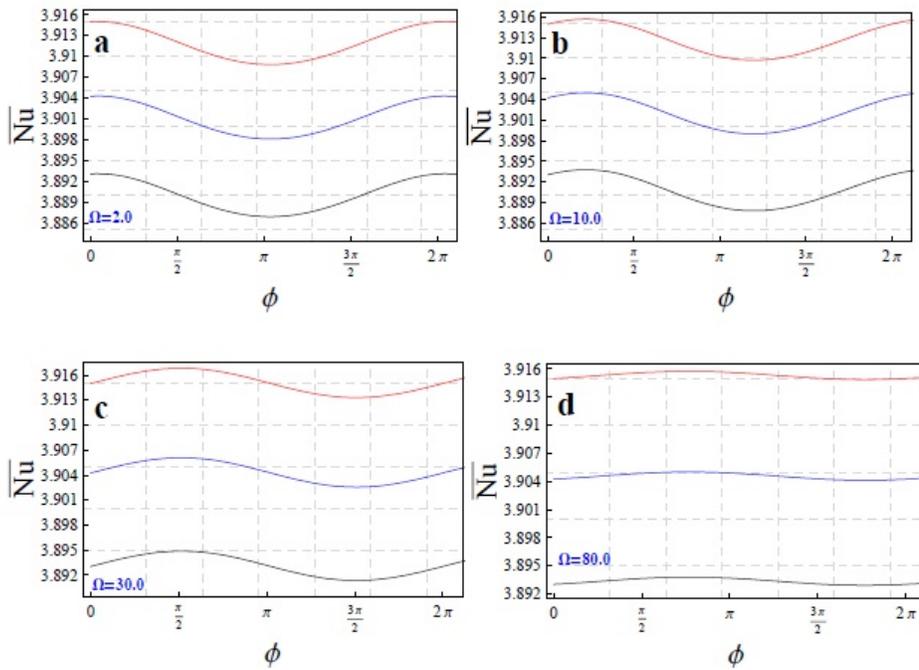


Figure 5. The frequency of modulation effect on mean nusselt number

7. Conclusions

The effects of throughflow and temperature modulation have been investigated in a two component viscoelastic fluid saturated porous medium by performing a weakly nonlinear stability analysis resulting in the complex Ginzburg–Landau amplitude equation. Depending on the above study the following conclusions are made:

1. Effect of solutal Rayleigh number Rs is to enhance the heat and mass transport.
2. Effect of relaxation time λ_1 is to enhance, retardation time λ_2 is to decrease the heat/mass transport.
3. The diffusivity ratio Γ decreases the heat/mass transfer as its value increases.
4. An increment in the amplitude δ of modulation is to enhance the heat/mass transfer.
5. The frequency Ω of modulation decreases the heat/mass transfer as its value increases.
6. Oscillatory mode ($\omega \neq 0$) enhances heat/mass transfer than stationary mode ($\omega = 0$).

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Appendix

The coefficients in Eq.(5.9) are defined as:

$$\begin{aligned} a_1 &= \lambda_1 \lambda_2, \\ a_2 &= 1 + (\lambda_2 - \lambda_1)c + \lambda_1 \lambda_2 c^2 \Gamma^2 - \frac{4\pi^2 Rs \lambda_1^2 a^2 (1 - \Gamma)}{(4\pi^2 - (\frac{Pe}{\Gamma})^2)}, \\ a_3 &= c^2 \Gamma^2 (1 + (\lambda_2 - \lambda_1)c) - \frac{4\pi^2 Rs a^2 (1 - \Gamma)}{(4\pi^2 - (\frac{Pe}{\Gamma})^2)}. \end{aligned}$$

The expressions given in Eqs.(5.16)-(5.17), one can simplify as:

$$\begin{aligned} Nu(s) &= 1 + \left(\frac{2\pi^2 a^2 c}{(4\pi^2 + Pe^2)(c^2 + \omega^2)} + \frac{2\pi^4 a^2}{(4\pi^2 + Pe^2)\sqrt{4\pi^4 + \omega^2}\sqrt{c^2 + \omega^2}} \right) |A(s)|^2, \\ Sh(s) &= 1 + \left(\frac{2\pi^2 a^2 c}{(4\pi^2 + (\frac{Pe}{\Gamma})^2)(\Gamma^2 c^2 + \omega^2)} + \frac{2\pi^4 a^2}{(4\pi^2 + (\frac{Pe}{\Gamma})^2)\sqrt{4\pi^4 \Gamma^2 + \omega^2}\sqrt{\Gamma^2 c^2 + \omega^2}} \right) |A(s)|^2. \end{aligned}$$

The expressions given in Eq.(5.18) are:

$$\begin{aligned} R_{31} &= -\lambda_2 \frac{\partial}{\partial s} (\nabla^2 \psi_1) - R_0 \lambda_1 \frac{\partial}{\partial s} \frac{\partial T_1}{\partial x} - R_2 \left(\lambda_1 \frac{\partial}{\partial \tau} + 1 \right) \frac{\partial T_1}{\partial x} + Rs \lambda_1 \frac{\partial}{\partial s} \frac{\partial S_1}{\partial x}, \\ R_{32} &= \frac{\partial \psi_1}{\partial x} \frac{\partial T_2}{\partial z} - \frac{\partial T_1}{\partial s} + \delta f_1(z, s) \frac{\partial \psi_1}{\partial x}, \\ R_{33} &= \frac{\partial \psi_1}{\partial x} \frac{\partial S_2}{\partial z} - \frac{\partial S_1}{\partial s}. \end{aligned}$$

The coefficients given in Eq.(5.19) are:

$$\begin{aligned} \gamma_1 &= \lambda_2 c + \frac{4a^2 \lambda_1 \pi^2 Rs}{(4\pi^2 + (\frac{Pe}{\Gamma})^2)(c\Gamma + i\omega)} - \frac{4a^2 \lambda_1 \pi^2 R_0}{(4\pi^2 + Pe^2)(c + i\omega)} + \frac{4\pi^2 R_0 a^2 (1 + i\omega \lambda_1)}{(4\pi^2 + Pe^2)(c + i\omega)^2} - \frac{4\pi^2 Rs a^2 (1 + i\omega \lambda_1)}{(4\pi^2 + (\frac{Pe}{\Gamma})^2)(\Gamma c + i\omega)^2}, \\ F(s) &= \frac{4\pi^2 R_0 a^2 (1 + i\omega \lambda_1)}{(4\pi^2 + Pe^2)(c + i\omega)} - \frac{2R_0 a^2 (1 + i\omega \lambda_1)}{(c + i\omega)} \delta I_1 \text{ where } I_1 = \int_0^1 f_2 \sin^2(\pi z) dz. \end{aligned}$$

$$k_1 = \frac{a^4 \pi^2 c R_0 (1 + i\omega \lambda_1)}{(4\pi^2 + Pe^2)(c^2 + \omega^2)(c + i\omega)} + \frac{a^4 \pi^4 R_0 (1 + i\omega \lambda_1)}{(4\pi^2 + Pe^2)(2\pi^2 + i\omega)(c + i\omega)^2}$$

$$- \frac{a^4 c \pi^2 R_s (1 + i\omega \lambda_1)}{(4\pi^2 + (\frac{Pe}{\Gamma})^2)(\Gamma^2 c^2 + \omega^2)(\Gamma c + i\omega)} - \frac{a^4 \pi^4 R_s (1 + i\omega \lambda_1)}{(4\pi^2 + (\frac{Pe}{\Gamma})^2)(2\pi^2 \Gamma + i\omega)(\Gamma c + i\omega)^2}.$$

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